Sample project topics Math 303

Bear in mind: The quality of a project is directly related to its having **substantive content** and a **well-defined**, **narrow focus**. The following are intended to be **suggestive**. Most topics need a narrowing of focus.

- Explore the evolution of the concept of symmetry. Can you say where and how it originated? If not, why not? What influenced its development? (Maybe select a particular period and place—ancient Greece, Rome, Persia, the Orient, the Renaissance, enlightenment or romantic thought.) Take an interdisciplinary point of view.
- **Evolution and symmetry.** What might the symmetry of a biological form have to do with its being fit for natural selection? Is there an advantage/disadvantage to being symmetric? At the macroscopic level? What about microscopic symmetry?
- Explore the symmetry or asymmetry of microscopic organisms. How does an organism's symmetry or lack thereof influence its development and function?

  For example, consider the structure of viruses.

  Reference: B. Voyles, *The Biology of Viruses* (1993).
- A "flat-fish" such as halibut or flounder starts out with a vertical orientation and, at some point, makes a one-quarter *turn* and takes on a horizontal position. (The eyes gradually move so that both are on top.) But, *which way* does the fish turn? To the left or right? Is the choice of direction genetically encoded or is it random? Can there be biological reasons for preferring left or right? Are there other examples of this kind of symmetry-breaking? For inorganic structures?

  Reference: Policansky, *The asymmetry of flounders.* Scientific American 246 (May 1982), 116-122.
- How symmetrical is a face? Take portraits of members of various species and compare the whole-face with one obtained by reflecting one-half of the face. Ponder what it might mean to describe something's being "close to symmetric".
- Make a film of two people one of whom hands something to the other—or interact in some way. Can an observer tell whether it's running forward or backward? If not, how could the symmetry be broken? What are the symmetries here?
- Compose a piece of music that exhibits some sort of symmetry. What sort? Can we hear symmetry? How symmetric can music be?
- One of the most common examples of symmetry (albeit on a microscopic scale) is found in crystals. What is the mathematical description of a crystal? Some examples are snowflakes, diamond, and pencil lead (graphite). Study an example and describe how a mathematical understanding of the crystal is valuable for understanding its properties. How has a mathematical theory of symmetry influenced crystallography?

- **God's favorite number.** Find examples of fivefold symmetry in nature. Is there some physical, biological, etc. connection between these cases? Speculate on what physical, biological, etc. purposes this common form of symmetry plays.

  Reference: Hargittai, *Fivefold Symmetry*
- Explore and describe the relationships between the lengths of various pieces of the pentagram (five-point star).

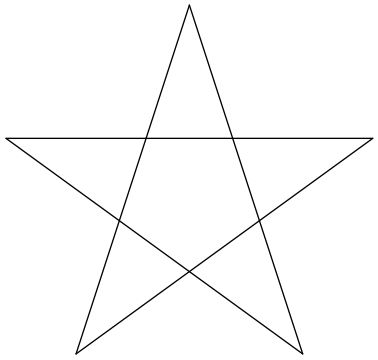

Figure 1: The pentagram

- Build and discuss a "polyhedral kaleidescope." This is made by joining several mirrors together in such a way that the "mirror images" produce the appearance of a whole regular polyhedron.
- A significant piece of the symmetry puzzle concerns asymmetry—the absence or breaking of symmetry. Explore a process that changes symmetry—that is, where the symmetry of the output is different from the symmetry of the input. An important consideration here is Curie's principle regarding symmetries of causes and effects. (See M. Golubitsky and I. Stewart, *Fearful symmetry.*)

  - Drop balls of paint onto paper and describe how the symmetry of the ball breaks. Can we predict what should happen?
  - Investigate the differences between ecosystems on north-south slopes of a mountain or valley. Is there symmetry within this asymmetric situation?
  - Investigate symmetry on the atomic and subatomic levels. consider how the symmetry of matter and antimatter is broken. What are the effects of such symmetry-breaking?

- **Anthropological symmetry.** Comparatively investigate how symmetrical designs, patterns, tools, etc. arise in various cultures. Are there universal forms that appear across cultures? What does a type of symmetry (or its absence) reveal about the social, cultural, and intellectual structures that produce it (or not).

  Source: D. Washburn. *Perceptual Anthropology: The Cultural Salience of Symmetry.* American Anthropologist (Sept 1999)

- Are there artistic (graphic, music, dance) analogues of the CP or CPT symmetry of particle physics? (Consider Magritte's painting *Interrupted reproduction.*)
- Consider the chiral structure of the narwhal's tusk. What accounts for its asymmetry? How does Curie's principle apply to this case? (See M. Golubitsky and I. Stewart, *Fearful symmetry* for a discussion of Curie's principle.)
- Life as we know it emerged as a change from symmetrical chemical structures to asymmetrical ones (amino acids, proteins, nucleic acid). Is this a violation of Curie's principle?

  Reference: Gardner, *The New Ambidextrous Universe*
- Make a spherical version of the Yin-Yang form, but with three regions in three distinct colors (no reflective symmetry). Can it be made so that after reflection and color exchange, the resulting sphere is superposable on the original?
- Why aren't rivers bilaterally symmetric? What's the source of the symmetry breaking?
- A highly significant problem for pharmaceutical manufacturers concerns the molecular chirality of a drug. Examine the issue of a drug and its mirror image. What makes this a commercial problem and what's involved in solving the problem?

  Reference: Thall, *When drug molecules look in the mirror.* Journal of Chemistry 73 (1996), 481-484.
- **Genetic symmetry.** Under mutation, an organism's genetic code is altered, but the double helix structure of the gene is preserved. How is this symmetry crucial to biological evolution?
- Let a configuration of particles—such as a gas—diffuse in a two-dimensional container. Start with the particles in a symmetric state (triangle, square, hexagon, etc.) and let them diffuse radially and then rebound from the the boundary. Does the resulting configuration of particles maintain its symmetry? How does the preservation or loss of symmetry depend on the shape of the boundary? Can some of the symmetry survive after rebound, but not all? It might be a good idea to begin with a small number of particles that are mathematical points. You might consider cases where the particles interact and don't interact with each other. The process could be simulated in a computer model—using a computer algebra system, for instance.
- Explore the idea that symmetry can be used to define a property/quantity/etc. For example can the energy of a system be characterized as a quantity that remains unchanged when the system's elements interact? Does the type of interaction allowed have to be constrained? Can such a description lead to useful results?
- Using a recursive model of growth, generate an "organism" that exhibits some type of symmetry. Does the symmetry change when passing from one generation to the next? Can you define a model for which the symmetry remains the same—at least from some point on—as the organism grows? (The "symmetry of symmetry.")