

Essay 1

Math 303

Spring 2017

Deadline: 12N Friday 24 Feb

Late deadline: 10 March

## Writing

- Select **two** of the following items and write a 500-600 word essay for each in response. Address the issues posed, but feel free to extend their scope.
- Using *concise and clear sentences*, incorporate symbols and illustrations into your text. Have an audience in mind. Focus on *developing* an explanation or argument. Using specific examples to illustrate a general idea or claim is often a helpful tactic. What are you trying to show and how are you're trying to show it?
- Writing well is difficult and can be painful. For writing assistance, consult the style and content guide at the class website. There's no required stylistic format; writing can range from technical to literary.
- Typed work is expected. Hand-drawn figures are acceptable.
- Submit in hard copy.

1) **Concrete or ideal?** Aristotle states that it's "impossible for mathematical objects to exist *in* sensible things." Is symmetry something that *exists in* an object? Or is it something *ideal* that we ascribe to or impose on an object? Does an object's symmetry depend on the space or setting that it's in?

2) **What a mirror does and doesn't.** How is it that a *flat* mirror reverses left and right, but not up and down? Can there be a mirror that reverses up and down, but not left and right? Reverses neither up and down nor left and right? Reverses both up and down and left and right?

3) **Changing symmetry.** Add features to a square—without changing the basic shape—in such a way that it has *exactly* four rotational symmetries; that is, the modified square has no reflective symmetries. Now, further modify it so that it has exactly two. Can you modify it so that it has exactly three rotational symmetries? Explain why or why not.

Modify the square so that it has reflective symmetry alone—other than the trivial symmetry. How many such symmetries—where the object has only reflective symmetries in addition to the trivial symmetry—can the modified figure have? Explain.

4) **Decreasing and increasing symmetry.** Consider an equilateral triangle—edges have the same length. There are six symmetries of the triangle if we use rotations and reflections. Suppose we "mark" the triangle with a line segment that coincides with one of the triangle's lines of reflective symmetry. What symmetries does the *new figure*—triangle-with-segment—have? Can you place a segment *across* a circle to produce a figure—circle-with-segment—that has exactly the same symmetries as the triangle-with-segment?

If we put in a second line segment of the same type, which symmetries does the resulting figure have? Can you place two segments *across* a circle to produce a figure—circle-with-two-segments—that has exactly the same symmetries as the triangle-with-two-segments?

Now, place the third such line segment on the triangle. What are the symmetries of this object? Can you place three segments *across a circle* to produce a figure—circle-with-three-segments—that has exactly the same symmetries as the triangle-with-three-segments?

Explain why the symmetry changes as the figure is modified.

- 5) **Decreasing and increasing symmetry II.** Take an equilateral triangle and attach equilateral triangles to each edge. Now close up the shape by bringing the three triangles together so that they meet at the point of a pyramid. This shape is called a regular tetrahedron.
- Describe *all* of the tetrahedron's rotational symmetries.
  - Describe the rotational symmetries of the structure you get when one of the edges is removed. Does it matter which edge?
  - After removing one edge, which additional edge should you remove in order to obtain an object with the *most* symmetry?
  - How would you remove three edges in order to produce an object with maximal symmetry? Can you achieve the result by removing one more edge from the object you obtained after removing two edges?
  - Discuss the same issues in the cases of removing four, five, and six edges.
- 6) Consider the valley pictured below. What sort of symmetry is *suggested* by the form of the hills? In what way or ways is the suggested symmetry broken? What's responsible for the symmetry-breaking and does Curie's principle apply? Can you imagine a circumstance under which the symmetry isn't broken?



- 7) **A universal mirror?** If the universe were reflected in a mirror, would you be able to tell that it had? Note that you get reflected as well. You might compare this case to one where the universe is rotated.