Introduction

What does the seventeenth-century Rundetarn (Round Tower) of Copenhagen have in common with the thirteenth-century Leaning Tower of Pisa? Or Houston's Astrodome, the first indoor baseball stadium built in the United States, with the vast dome of the Pantheon in Rome? Or a Chinese pagoda (fig. 1) with the Sydney Opera house (fig. 2)? A first response might be "shape" but a more accurate answer would be "symmetry". Each of these strange couples of buildings share a different kind of symmetry that links them, in spite of their temporal and cultural differences. As Magdolna and István Hargittai have noted, symmetry, in architecture as in other arts, is "a unifying concept".[1]

Architecture, as any compositional art, makes extensive use of symmetry. Across all cultures and in all time periods, architectural compositions are symmetrically arranged. There are so many kinds of symmetry, so many kinds of architecture, and so many ways of viewing architecture, that the argument threatens to become so generalized that it loses all meaning. The general exposition of symmetry types found in architecture has been admirably presented in recent work.[2] While I wish to review symmetry types in architecture briefly in order to provide as wide an overview as possible within the limits of this paper, my ultimate object is to explore why an architect might choose a given symmetry type, and thus to provide insight into the design process from the point of view of symmetry.

The special case of architecture

Architecture differs fundamentally from other arts because of its spatiality. Identifying a type of symmetry in a two-dimensional composition is relatively straightforward; the identification of symmetry types in a three-dimensional object such as a sculpture is somewhat more complicated because our perception of the object changes as we move around it. In the case of architecture, we not only move around it, but we move through it as well. This means that architecture provides us with a special opportunity to experience symmetry as well as to see it. This is possible because architecture consists of two distinct components: solid and void. Architecture is most frequently characterized by the nature of its elements: we recognize a Greek temple by its portico and pediments; a Gothic cathedral is characterized by its pointed arches and flying buttresses. These are the elements that make up the solid component of architecture, and it is likely that it is with this solid component the lay person has the most experience. Naturally in the composition of these elements that one would expect to find various kinds of symmetry relations, and this, the symmetry that we see, is what I will be examining in the first part of this paper.

On the other hand, all these solid elements constitute an envelope around what we experience when we move through a building, that is, the void, or architectural space. In a very real way, the true work of the architect is to shape the void, which becomes the theater of the actions that take place in the building. This architectural space is most likely characterized by symmetry as well, though it is perhaps less familiar, and it is a
symmetry which we experience. This is what I will examine in the second part of this paper.

**An Overview of Symmetry types in Architecture**

Symmetry types are divided into two categories: point groups and space groups. Point groups are characterized by their relationship to at least one important reference point; space groups lack such a specific reference point. Both point groups and space groups are found in architecture.

**Bilateral symmetry** is by far the most common form of symmetry in architecture, and is found in all cultures and in all epochs. In bilateral symmetry, the halves of a composition mirror each other. It is found in the facade of the Pantheon in Rome; some 1700 years later on a continent undreamed of when the Pantheon was built, we find the same symmetry in the mission-style architecture of the Alamo in San Antonio, Texas. Bilateral symmetry is present also not only on the scale of a single building, but on an urban scale. An example of this is found in the design of the PraHo do Comercio in Lisbon, Portugal, where three urban elements (a major public square, a monumental gate and the wide commercial street beyond the gate) are symmetrical with respect to a long horizontal axis that governs our visual perspective.

The popular of bilateral symmetry is probably an expression of our experience of nature, and in particular with our experience of our own bodies. As many cultures believe that God created man in His own image, architecture has in turn probably been created in the image of man. Not all bilateral symmetry is of equal value in architecture, however. Two schemes for facades are shown in fig. 3. In one, there are an unequal number of bays; in the other, there are an equal number of bays. The first is an example of "orthodox" bilateral symmetry, where the facade is divided into two equal halves; but in the second, the axis of symmetry that divides the facade into two equal and independent halves creates a dualism. If it is true, as Dagobert Frey maintains, that bilateral symmetry represents "rest and binding"[3], then dualism represents divisibility. Traditionally, dualism in architecture has been considered something to be avoided. The temples of ancient Greece, for example, always had an even number of columns so that there was never a column on the central axis of the facade. The avoidance of the dual by classical architecets probably stems from the ambiguity frequently attributed to the number 2, regarding with suspicion from the time of Pythagoras. The number 2 was considered untrustworthy (a female number) because it could be divided into halves, in contrast to the number 3 (a male number) which was not divisible into two parts. Even in modern architectural theory, dualism in architecture is considered a "classical and elementary blunder" and identified with the "amorphous or ambiguous".[4] These reservations not withstanding, dualism does exist in architecture. The fourteenth-century Oratory of Orsanmichele in Florence is an example (fig.4). It has a dual function: an oratory on the ground floor and a granary above. It has an unusual two-aisled plan. It has two altars. The difficulty of the dual on the level of architectural experience is best exemplified by the problem of the two altars. Where does one stand in the church? One is forced to make a decision whether to stand in front of one altar or the other. It is comparable to a house with two front doors. Where is the entrance? Usually this kind of decision is made for the spectator by the architect, who places one altar in a central position, or one prominent front door on the facade of a house. Thus, dualism in architecture presents a kind of a challenge to both the spectator and the architect.

**Rotation and reflection** provide a sense of movement and rhythm in architectural elements and an emphasis on the central point of the architectural space. The Sacristy of the basilica of S. Spirito in Florence, designed by Giuliano da San Gallo in the last years of the fifteenth century, is octagonal
in plan and both the architecture and the distinctive pavement design exhibit rotational and reflection (fig. 5). Domes, whether hemispherical such as that of the Pantheon or octagonal such as the great cupola of the Cathedral of Florence designed by Filippo Brunelleschi, also exhibit both rotation and reflection.

**Cylindrical symmetry** is that found in towers and columns. Verticality in towers represents a defiance of gravity. Rare examples of spherical symmetry may also be found in architecture, though the sphere is a difficult form for the architect because human beings move about on a horizontal plane. The project for a cenotaph for Isaac Newton, designed by Etienne-Louis Boulée in 1784, demonstrates spherical symmetry.

**Chiral symmetry** is perhaps less well-known than other types of symmetry but frequently effectively used in architecture. Chiral symmetry is found in two objects which are each other's mirror image and which cannot be superimposed, such as our hands. The two opposing colonnades designed by Gianlorenzo Bernini that surround the elliptical piazza in front of St. Peter's in Rome exhibit chiral symmetry (fig.6). In Budapest, the two Klotid Palaces that tower above Felszabadul<sup>5</sup>'s Square, each with asymmetrically placed towers and facade embellishments, are examples of chiral symmetry. A very subtle form of chiral symmetry is presented by the two leaning towers of the newly-completed Puerta de Europa in Madrid, designed by architect John Burgee in collaboration with Philip Johnson. Chiral symmetry in architecture is another way to place visual emphasis on the central element of a composition. In the case of the Puerta de Europa, for example, the two inclined towers emphasize the broad boulevard that passes between them, aptly forming a "gateway to Europe".

**Similarity symmetry** is currently receiving a great deal of attention and is best known for its identification with fractals. Similarity symmetry is found where repeated elements change in scale but retain a similar shape, such as in the layered roofs on a pagoda (see fig. 1 above), the forms of which diminish in size but retain their form as they get closer to the top of the building. Another example of similarity symmetry is found in the nestled shells of the Sydney Opera House, designed by Joern Utzon in 1959 (see fig. 2 above). The shells are all segments of a sphere, thus similar in shape while differing in size and inclination. Another example of similarity symmetry is found in the Castel del Monte in Apulia in Italy, built by Friedrich II at the end of the first millennium. The basic form of the octagonal outer walls of the fort is repeated at a smaller scale in the interior courtyard, and again in the smaller towers which are added to each apex of the main octagon.[5] Similarity symmetry is also often used where it is least obvious, as in the relationships between room sizes. Frank Lloyd Wright used a kind of similarity symmetry in his design for the Palmer House in Ann Arbor, Michigan, in 1950-51.[6] In this case, Wright chose an equilateral triangle as a planning module, repeated at a number of levels and sizes to organize the design of the house. Similarity symmetry, whether visually apparent or not, results in a high degree of order within an architectural design, and lends unity to a composition.

**Spiral or helical symmetry** may be thought of as a special kind of similarity symmetry. Helixes and spirals in architecture often represent continuity. In spiral staircases, the unbroken form expresses the continuity of space from level to level throughout the building. In the fantastic twisted spires of Copenhagen or of Borromini's S. Ivo alla Sapienza in Rome, the theme of continuity is expressed by the unbroken upward progression of the form. Frank Lloyd Wright used the helix in his 1946 design of the Guggenheim Museum of New York. In this case, the exterior of the building reflects the form of the giant helical ramp on the interior. The gallery spaces are arranged along one side of the ramp. The museum visitor takes the elevator to
the top floor of the space, then spirals his way down the ramp to the bottom, admiring the art on display along the way. Criticism of the building focused on the fact that the downward spiral forced the visitors to hurry through the museum, unconsciously rushed by the pull of gravity. Legend has it that Wright, who placed greater value on architecture than on art, deliberately designed the building in order to get the visitor out as quickly as possible! In reality, however, the helical ramp once again expresses spatial continuity.

**Translational symmetry** falls in the category of space group symmetry, and is, after bilateral symmetry, the most common kind of symmetry found in architecture. Translation of elements in one direction is found in solemn rows of soldier-like columns, or in the springing succession of arches in an aqueduct. Translation of elements in two directions is present in the wallpaper-like patterns of the curtainwall facades of many modern buildings. Translation may also involve the repetition of entire pieces of buildings, especially in our own century, and may be one reason by modern architecture is so often referred to as boring or monotonous. Translational symmetry seems to carry with it an emphasis on a superlative quality in architecture: the longest, the broadest, the tallest.

This concludes my survey of types of symmetry found in arrangements of architectural elements. For the architect, the knowledge of symmetry types is a powerful tool, for it provides him not with a means for precisely describing a building, but with a range of expressive possibilities. We will learn more about the expressive possibilities of symmetry when we look at the use of symmetry in architectural space. However, before turning to this, I should emphasize another aspect of symmetry in architecture that makes it a special case in the study of symmetry.

**Multiple Symmetries in Architecture**

In choosing the examples of various symmetry types for the previous section of this paper, I purposely focused on one aspect or part of a building that exhibits a single kind of symmetry. However, in most buildings we find more than one kind of symmetry. For example, in the Chinese pagoda, we can see at the same time both the cylindrical symmetry inherent in the building's organization about the vertical axis, and the similarity symmetry of the diminishing sizes of the layered roofs. A colonnaded temple facade may demonstrate bilateral symmetry, but it also demonstrates translation. These are examples of multiple symmetries that can be observed without requiring us to change our viewpoint of the building. We also perceive multiple symmetries when we change our position relative to the building, as for example, when we move from outside to inside. Domes are a very good example of this. From the outside, domes appear to be organized about a vertical axis (as they indeed are). When viewed from the inside, however, they appear to be organized about a central point.

Multiple symmetries also arise when a building is composed of multiple elements, some or all of which having its own symmetry. The symmetry type that we identify at any given moment, then, is a result of our physical position in relation to the building. It is important to make this point about multiple symmetries, because most architecture of any complexity at all is designed as a series of spaces that are meant to be experienced sequentially, as though the architect is telling us a story. Changing symmetries can be as important to the unfolding of the story as any of the other devices an architect has at his service. A closer examination of the Pantheon will illustrate the experience of an architectural "story".

The Pantheon in Rome is an excellent example of the experience of multiple symmetries that is common in architecture. When we stand in the piazza in front of the Pantheon, we notice right away the bilateral symmetry of the principal facade. Moving around the building, we discover that the Pantheon is composed of three easily-identified elements: the columned porch, a small intermediate block, and the great rotunda (fig. 7). The three are arranged in sequence: here is the beginning of the "plot" of the story. As we enter the Pantheon, we see that the three elements are arranged with respect to a common horizontal axis; it is this axis that gives rise to the bilateral symmetry. However, once inside, the horizontal axis that we have followed to gain entrance into the rotunda disappears. It is replaced by a vertical axis that runs from the center of the pavement up to and through the oculus of the dome above. Thus the dominant symmetry is no longer
bilateral. The lower zone exhibits cylindrical symmetry, while the hemispherical dome above exhibits rotation and reflection. The reason for the change in symmetries is that, when we enter into the rotunda we leave behind the zone of the terrestrial, represented by the horizontal axis, and experience the zone of the celestial, symbolized by the vertical axis. The Pantheon is a temple dedicated to all the gods; the universe itself is represented in the rotunda by the form of the sphere, half of which is actually present in the coffered cupola which crowns the space, while the other half is only made implicit in the proportions of the space (as mentioned before, the sphere is problematic in architecture because human beings require a horizontal plane). The sphere contains an infinite number of planes of reflection and rotation; its infinity symmetry makes it an apt symbol for the cosmos.

Symmetry in Architectural Space

Having examined how symmetry is found in the parts of a building that we see, we may take a look at how symmetry relates to the part of the building we don't see, which is the void that is the architectural space. Two concepts are fundamental in describing architectural space: center and path. Center relates to a single important place within the larger architectural space, such as the altar in a church. Path relates to the spectator's movement through the space. Christian Norberg-Schulz writes that "...centre and path are present in any church, but their relationship differs." This relationship actually determines how we perceive the architectural space of any given time period. In terms of symmetry, center may be thought of as "point" and path, as "axis." The following, very brief, examination of some 1500 years of architectural history hopes to demonstrate that as architectural space evolved through the centuries, so did the dominant symmetries.

In Roman architecture, strictly observed axial symmetry gives rise to spaces that are monumental and static, that is, generally embodying a sense of equilibrium rather than expressing a sense of dynamic movement.

Consider the symmetry relations of the plan of a Roman basilica, a secular building type used as a court of law (fig.8). It is rectangular, with an apse on each end of the major axis and a doorways on each end of the minor axis. The architectural elements are always arranged so that like elements are always opposite: apse to apse, column to column, doorway to doorway. Excavations have brought to light the remains of the pavements used in basilicas; they underline the sense of balance and equilibrium that characterize the architecture, as frequently they are based on patterns described by translational symmetry in two directions, rather than by any other kind of more dynamic symmetry type such as rotation. This same static arrangement of architectural elements is found in the rotunda of the Pantheon, Rome. Here the plan is a circle, with eight reflection planes and one four-fold axis of rotation (to be precise, the symmetry is approximate because the entrance is opposite a large round apse). Again we find oppositions: apse to apse, aedicule (a canopied niche flanked by colonnettes) to aedicule, niche to niche, column to column. The strict axial symmetry establishes
the sense of equilibrium within the space that is characteristic of Roman architecture. It is interesting to note, however, that neither the axes nor the center point is made explicit through the pavement design of the rotunda, which is like that of the basilica based upon translation in two directions. Thus the symmetry was an organizing device for the architecture, but does not determine the movement of the spectator within the space. This is one characteristic that distinguishes Roman architecture from that of later periods, in which we will see how axes and centers are used to provide a specific dynamic emphasis and encourage movement. After the legalization of Christianity in the fourth century, Christian architects chose to adapt the Roman basilica to their own ecclesiastical needs. To do so, they removed the entrances from the minor axis and placed a principal entrance on one end of the major axis, placing the altar in the remaining apse (fig. 9).[9]

Thus the symmetry was radically altered, there remaining only a single plane of reflection and no planes of rotation: the plan of the Christian basilica is bilaterally symmetrical. The axis of symmetry takes on an all-important symbolic role: it becomes a path, symbolizing the earthly pilgrimage of the Christian making his way towards the Kingdom of God. The pavement designs of many of these churches make explicit the axis that governs the architecture. Bilateral symmetry is favored over all other symmetry types during the Early Christian, Romanesque and Gothic periods, spanning from 300 to 1300 AD, because it best expressed the Christian ideal. It is the necessity of expressing the concept of pilgrimage, and not only that of expressing order as suggested by Hermann Weyl, that gave rise to the bilateral symmetry that dominated Christian architecture up until the Renaissance.[10] In addition to bilateral symmetry in plan, the sense of movement along a path is underlined by the translation of elements in a horizontal direction parallel to the dominant longitudinal axis. It is this kind of dynamic indication of direction that is lacking in Roman architecture. As architectural and philosophical ideals changed in the Renaissance, so did the type of symmetry most frequently used. Sacred architecture was intended as a model of the cosmos created by God. To this notion, Humanism added the concept that, because man is God's most important creation in the cosmos, his place is in its center. The centrally-planned building was favored as best reflecting the perfection of the cosmos, thus rotational and reflectional symmetries were particularly favored during this period. The center point is usually made explicit in the pavement design: this particular emphasis on the center point induces the spectator to place himself there.

Pavement designs from the fifteenth, sixteenth and seventeenth centuries use rotation, reflection and similarity symmetry to emphasize the center. The rosette is a motif that often appears in pavement designs of this time, as for example, in the octagonal Sacristy of the basilica of S. Spirito in Florence (see fig. 5 above). Here the rosette is formed from sections of a logarithmic spiral. To create the curvilinear checkerboard motif, a logarithmic segment is rotated a given number of times about the center in one direction, forming a fan pattern, then the direction of the segment is reversed and rotated about the center the same number of times in the opposite direction. The resulting rosette pattern has modules that increase in size but maintain their proportional similarity as they move farther from the center, and is therefore characterized by similarity symmetry as well as rotation and reflection. Another example of paving patterns from this period may be seen in the Cathedral of Florence, S. Maria del Fiore, in which trapezoid-shaped modules increase in size as they move away from the pattern's center, again demonstrating reflection, rotation and similarity symmetry.
These patterns were no doubt favored because the perspective illusion they create is an excellent means of emphasizing the central point of the design, and through this, the central point of the architectural space. Thus we see that in this arc of architectural history, the dominant symmetry evolved from a generalized axial symmetry in the Roman age, to bilateral symmetry in the Paleo-Christian, Romanesque and Gothic ages, to rotational and reflectional symmetry in the Renaissance. Our recognition of the symmetry in an architectural space is one step towards our understanding of the architecture, a means we are given to interpret the architectural "story" we are experiencing.

Conclusions
At this point I draw to a close this discussion of architecture and symmetry. I hope that the wide variety of symmetry types and their various combinations as well as the use of symmetry to define space has been made clear. However, the topic of symmetry in architecture is far from exhausted. There are some further aspects of the subject that I am now studying but about which I am not yet in a position to draw conclusions, and for which this present paper forms a background.

One of these aspects has to do with "broken symmetries". The Pantheon in Rome provides one example of a symmetry break: the cylindrical lower zone of the rotunda is characterized by four planes of reflection and fourfold rotation, while the hemispherical dome above is characterized by twenty-seven-fold rotation. Four and twenty-seven have no common divisors, thus the symmetry "break." Another example of broken symmetries is found between horizontal tiers of the Baptistery of Pisa, which are alternately based on rotations of twelve and twenty.[11] These of course, are historical examples. Many other examples are present in modern architecture.

A second, very important question concerning the architecture today is this, "Why have contemporary architects deliberately chosen to disregard traditional types of symmetry in their architecture? The designs of Richard Meyer and Frank Gehry in the United States come to mind. The advantage of examining contemporary architecture lies in the fact that the architects are most often still living, and while we can never ask the architect of the Pantheon why he broke the symmetry of the rotunda, we can ask Frank Gehry why the design of Guggenheim Museum in Bilbao apparently throws a consideration of symmetry to the wind. I say apparently, because I would want to ask the architect for an explanation before hazarding any judgement of my own. So I hope in a future paper to be able to present the fruits of this current research, and shed even more light on the uses of symmetry, both apparent and otherwise, in architecture.

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Notes


9. Ibid., 59. "La basilica romana è simmetrica rispetto ai due assi: colonnati contro colonnati, abside di fronte ad abside. Essa crea quindi uno spazio che ha un centro preciso ed unico, funzione dell'edificio, non del cammino umano. Che cosa fa l'architetto cristiano? Praticamente due cose: 1) sopprime un'abside, 2) sposta l'entrata sul lato minore. In questo modo, spezza la doppia simmetria del rettangolo, lascia il solo asse longitudinale e fa di esso la direttrice del cammino dell'uomo." (The Roman basilica is symmetric with respect to the two axes: colonnade opposite colonnade, apse opposite apse. This creates a space which has a precise and unique center, a function of the building, not of man's movement. What did the Christian architect do? Essentially two things: 1) suppressed an apse, 2) moves the entrance to the shorter side. Thus he breaks the dual symmetry of the rectangle, leaving only the longitudinal axis, which he makes the directrix of man's movement.--translated by Kim Williams.) return to text


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