

Essay

Math 309

Fall 2019

Deadline: class, 20 November

Late deadline: noon, 27 November

- Select **one** of the following items and write a 800-1000 word essay in response. Address the issues posed, but feel free to extend their scope.
 - Using *concise and clear sentences*, incorporate symbols and illustrations into your text. Have an audience in mind. Focus on *developing* an explanation or argument. Using specific examples to illustrate a general idea or claim is often a helpful tactic. What are you trying to establish and how are you're trying to establish it?
 - Writing well is difficult and can be painful. For writing assistance, consult the style and content guide at the class website. There's no required stylistic format; writing can range from technical to literary.
 - Submit **double-spaced in hard copy**.
- 1) Perhaps the standard model of a complex system involves emergent global properties arising from interactions based on rules that apply locally. Using a real or imagined system, discuss and illustrate how particles or agents that are far apart—in some sense that's meaningful relative to the system at hand—can exert an influence on one another. How does such an effect play into the properties and structures that emerge?
 - 2) Imagine two ice cream vendors working along a stretch of beach. They each pick an initial position to set up their carts. Describe a dynamical process that can result from the two vendors trying to gain an advantage over the other. Is there a rule (analogous to a map) that the vendors might follow? What outcomes follow from repeated (iterated) application of the rule? Are the outcomes reasonable? That is, do they agree with the way you would expect people to actually behave? How might the dynamics change if a third vendor appears?
 - 3) Frequently, an average of quantities—speeds, rainfall, heights—doesn't refer to something real. (Among a group of people, there needn't be someone whose height is the average height.) Describe a situation—it can be physical or virtual—in which an average of some set of quantities corresponds to a real physical condition or state. Be careful to quantify the constituent parts and elaborate on how the average value of these quantities constitutes the relevant condition or state.
 - 4) Carefully discuss how a dynamical process can be both deterministic *and* unpredictable. Using a specific example might be a good means of exposition.
 - 5) Compare the dynamical behavior of a financial market to that of the weather. Are either predictable? Are there constraints on their predictability? Are they deterministic? Do they tend to a stable equilibrium state?
 - 6) The “object” on which evolution acts is a population as a heritable variation possessed by individuals is selected due to some adaptive advantage that it confers. Can evolution select for a variation that's emergent—a change in a property that arises only collectively? Be careful to take account of how the emergent variation is heritable.
 - 7) At one level, competition and cooperation are modes of interaction among a population of agents (e.g., organisms in an ecosystem or participants in an economy). At another level, they are emergent properties of a population of agents. Discuss and illustrate how a property or behavior that's emergent feeds back into the way agents interact and so influences the nature of a system's emergent properties or behavior. Specifically, can competition promote cooperation and can cooperation promote competition? Characterize interactions that are competitive and cooperative.

- 8) Each member of a class selects a number of extra credit points (5 or 10) to be added to the final exam score. If 10% or more select 10, everyone receives 0. Otherwise, each person receives the selected amount. How do you choose? Is there a rational choice? Can you set up the process as a game—say, there are two members of the class? Three? Compare this dilemma to the use of the commons—say, energy resources, CO₂ emissions. (See G. Hardin, *The tragedy of the commons*, Science, 1968, 1243-1248.)
- 9) Evaluate the claim that a nation should manage its finances in the same way that a family or individual does. For instance, when times are tough, debt should be reduced or avoided. How does the number of agents influence the nature of the interactions among them? Are there properties or structures that emerge only in the large population?
- 10) Devise an evolutionary algorithm that attempts to solve an optimization problem—such as the polygon problem: given N line segments, what polygonal arrangement of the segments maximizes the enclosed area? You need to specify the protocols, the reproduction cycle, and the selection process (fitness??). It might help to work on the simplest case of a quadrilateral first and then extend the procedure. Discuss how well you think the algorithm would work.