Lab exploration 1: Dynamical systems
Math 309 Fall 2022
Deadline: 12 September

- Conduct experiments as directed.
- **Journal entry.** Respond to each of the "journal queries." Using *concise and clear sentences*, incorporate data, symbols, and illustrations into your text. Have an audience in mind. Focus on *developing* an explanation or argument that stems from your simulations.

  Submit 300-400 words 2-3 pages double-spaced to the class dropbox.
- **Recommended.** Work in groups of 2 or 3. Submit one journal entry for the group.
- *NetLogo* models are available either in the Models Library or on the course website.
- Text on the *NetLogo* interface appears in blue sans serif font.

Model: Tent map. The model presents a family of dynamical systems that you can iterate using a range of initial points. Setting a value for the parameter R specifies the map expressed by

$$T_R(x) = \frac{R}{2}(1 - |2\,x - 1|) \qquad 0 \le R \le 2 \qquad 0 \le x \le 1.$$

The point x0 specifies a starting point for the iteration. The setup button reads the set values of R and x0. The button labeled go computes the orbit

$$x_0,\ T_R(x_0),\ T_R^2(x_0),\ \ldots$$

and reports the data numerically, graphically, and as a time series (thinking of an iteration as a time step. The value of ss is used to scale the graph. For this lab, set it equal to 1.

1.1 Journal query.

Compute orbits for several initial points and for values of R close to 0. Briefly describe the orbit's behavior: attracted to a fixed point? to a cycle? does it wander? As R increases what happens to the rate at which the orbit develops? Does the dynamics make sense in light of a cobweb diagram?

Increase the value of R until the previous behavior stops. At what value of R does this change occur? Use a cobweb diagram to describe the dynamics when R is a little bit greater than this value.

1.2 Journal query.

Gradually increase R and look for *qualitative* changes in dynamical behavior. Describe one or two cases. Discuss how the time series plot changes as R increases.

1.3 Journal query.

Set $R = 2$ and plot several orbits. Discuss the behavior.

1.4 Journal query.

Now set $R = 1.99$ and plot some orbits. Compare the results to those for $R = 2$. What might explain the difference between the two cases?

Model: Logistic map. Motivated by ecological considerations the logistic family of maps has the form

$$L_R(x) = R\,x\,(1-x) \qquad 0 \le R \le 4 \qquad 0 \le x \le 1.$$

The model runs in the same way as that of the tent map. Run experiments similar to those done with the tent maps–noting that R varies from 0 to 4.

1.5 Journal query.

Describe the long-term behavior of orbits for values of R slightly **smaller than** 3 and then for values slightly **larger than** 3.

1.6 Journal query.

Find another value of R where the dynamical behavior changes qualitatively. Describe the change.

1.7 Journal query.

Compare the logistic family to the tent family. Briefly discuss some similarities and differences.