Lab exploration 2: Fractal systems
Math 309 Fall 2022
Deadline: 21 September

- Conduct experiments as directed.
- **Journal entry.** Respond to each of the "journal queries." Using *concise and clear sentences*, incorporate data, symbols, and illustrations into your text. Have an audience in mind. Focus on *developing* an explanation or argument that stems from your simulations.

  Submit 300-400 words 2-3 pages double-spaced to the class dropbox.
- **Recommended.** Work in groups of 2 or 3. Submit one journal entry for the group.
- *Net Logo* models are available either in the Models Library or on the course website.
- Text on the *NetLogo* interface appears in blue sans serif font.

Model: Tent map. Set the value of R equal to 3. Also, adjust the graph scaling parameter: ss = .67.

1.1 Journal query.

Select an arbitrary starting point and generate its orbit. Does the orbit remain inside the interval $[0, 1]$. Find a point whose orbit "escapes" from the interval. How many iterations does it take to escape? What happens to the orbit *after* it's escaped?

1.2 Journal query.

Now find a point whose orbit takes twice as many iterations to escape as the point that you previously found. Can you continue to find points with escaping orbits that require more/many iterations? Where's a good place to look for such points? (Take a look at the next journal query.)

1.3 Journal query.

Are there points whose orbits don't escape the interval? Obviously, yes. Take $x_0 = 0$. Find five more points whose orbits remain in $[0, 1]$. How many such points do you think there are? Try to describe them in a systematic way.

1.4 Journal query.

Make a sketch of the interval from 0 to 1 that indicates which points leave the interval after *one* application of the map. Now indicate which points leave after two iterations. (Coloring these sets could help.) Three iterations? Describe the *whole* collection of "escaping points." Is it a familiar object?

Model: Fractals. Run the Koch Curve construction for ten steps. Note the length of the "curve" as well as the fractal dimension.

1.5 Journal query.

Next, iterate the Dragon Curve construction for ten steps and note the dimension and length. Briefly describe the rule that's followed in order to generate the figure. Briefly explain why the fractal dimension of the dragon curve is a whole number, namely, 2.

1.6 Journal query.

Compare the lengths of the Koch and dragon curves for a given number of iterations. Does the rate at which the length grows affect the dimension? Briefly explain.