

# Lab exploration 7: Network systems

## Math 309 Fall 2016

### Deadline: 12N Friday 9 December

- Conduct experiments as indicated.
- **Journal entry.** Respond to each of the “journal queries.” Using *concise and clear sentences*, incorporate data, symbols, and illustrations into your text. Have an audience in mind. Focus on *developing* an explanation or argument that stems from your simulations.  
Submit 300-400 words 2-3 pages double-spaced in **hard copy**.
- **Recommended.** Work in groups of 2 or 3. Submit one journal entry for the group.
- **Suggestion.** Before running the simulations, read the “What is it?” and “How it works” sections under the [Info](#) tab.

**Model: Random Network.** (Location: class website.) You can generate a network where the edges are placed at random. [setup](#) produces a selected number of nodes and [wire3](#), with a chance of 50%, places [num-links](#) edges between randomly selected pairs of nodes.

#### 7.1 Journal query.

If a random network of this type has  $N$  nodes and  $E$  edges, what’s the average degree of a vertex? Set [num-nodes](#)=50. Beginning with [num-links](#)=100 and incrementing by 100 until reaching 1000, at each step consider the data that result (use an average of several different networks).

- a) What’s the expected average degree of a node? Check the calculated value against the values of maximum and minimum degrees ([max-deg](#) and [min-deg](#)).
- b) What form does the degree distribution take? How does it change as the number of edges increases? Is there any evidence that it obeys a power law?

**Model: Small Worlds.** (Location: class website.) Beginning with a “ring” of nodes, the model produces a network in which each node is connected to its two neighbors on each side. You can see how this works by setting up a 10-node network. The procedure allows you to “rewire” the network by randomly selecting a node  $v$ , removing an edge attached to  $v$ , and installing a new edge from  $v$  to a randomly selected node. [rewire-one](#) applies the process one node at a time while [rewire-all](#) replaces an edge at all nodes with a chosen probability. (Probability 0 rewires nothing and probability 1 rewires all nodes.)

Notice that the initial clustering coefficient is .5 for any number of nodes  $n$ . Check this result.

#### 7.2 Journal query.

What’s the relationship between  $n$  and the diameter?

#### 7.3 Journal query.

Begin with a 50-node network and rewire nodes one at a time. Notice how the clustering coefficient (CC), the average-path-length (APL), and the diameter change as the number of rewired nodes increases. Do the quantities stabilize? If so, determine when this occurs in terms of the fraction of rewired nodes. What are the approximate stable values? As a function of the fraction of rewired nodes, when does the diameter decrease most rapidly? Repeat the experiment for a 100-node network.

**Model: Preferential Attachment.** (Location: [Models Library/Sample Models/Networks.](#)) The simulation grows a network by adding nodes and edges according a simple rich-get-richer principle: the likelihood that a new node  $a$  will be connected to an existing one  $b$  is proportional to the *relative* degree of  $b$  (equals the ratio of the degree of  $b$  to twice the total number of edges in the current network).

#### 7.4 Journal query.

Run the simulation for at least 1000 time steps—producing as many nodes. Describe the evolution of the degree-distribution. When, in terms of the number of nodes, does power law behavior become apparent? Where in the distribution—that is, for what degree values—is there the greatest deviation from a power law? Explanation? Estimate the power law’s exponent.

#### 7.5 Journal query.

Devise a network experiment (for any type of network). Describe what parameters are involved and what sort of behavior is the object of investigation. Run the experiment and report the results.