Lab exploration 9: Network systems
Math 309 Fall 2022
Deadline: class 9 December

- Conduct experiments as indicated.
- **Journal entry.** Respond to each of the "journal queries." Using *concise and clear sentences*, incorporate data, symbols, and illustrations into your text. Have an audience in mind. Focus on *developing* an explanation or argument that stems from your simulations.
- Submit 300-400 words double-spaced to the class dropbox.
- **Recommended.** Work in groups of 2 or 3. Submit one journal entry for the group.
- **Suggestion.** Before running the simulations, read the "What is it?" and "How it works" sections under the Info tab.

Model: Random Network. (Location: class website.) You can generate a network where the edges are placed at random. setup produces a selected number of nodes and wire3, with a chance of 50%, places num-links edges between randomly selected pairs of nodes.

9.1 Journal query.

If a random network of this type has N nodes and E edges, what's the average degree of a node? Set num-nodes=50. Beginning with num-links=100 and incrementing by 100 until reaching 1000, at each step consider the data that result (use an average of several different networks).

a) What's the expected degree of a node? You can think of the expected degree of a node as the number that, when taken to be the *degree of each node*, will produce the total number of links. So, imagine that L links are attached to each node (i.e., each node has degree L). Use this assumption to count the number of links in the network and so, derive an equation that relates L to the number of nodes N and the total number of links E. Be careful of overcounting.

   Check the theoretical value against the values of maximum and minimum degrees (max-deg and min-deg). You can do this by taking the average of the max and min degrees.

b) What form does the degree distribution take? How does it change as the number of edges increases? Is there any evidence that it obeys a power law?

Model: Small Worlds. (Location: class website.) Beginning with a "ring" of nodes, the model produces a network in which each node is connected to its two neighbors on each side. You can see how this works by setting up a 10-node network. The procedure allows you to "rewire" the network by randomly selecting a node v, removing an edge attached to v, and installing a new edge from v to a randomly selected node. rewire-one applies the process one node at a time while rewire-all replaces an edge at all nodes with a chosen probability. (Probability 0 rewires nothing and probability 1 rewires all nodes.)

Notice that the initial clustering coefficient is .5 for any number of nodes N. Check this result.

9.2 Journal query.

At the start, what's the relationship between N and the diameter?

9.3 Journal query.

Begin with a 50-node network and rewire nodes one at a time. Notice how the clustering coefficient (CC), the average-path-length (APL), and the diameter change as the number of rewired nodes increases. Do the quantities stabilize? If so, determine when this occurs in terms of the fraction of rewired nodes. What are the approximate stable values? As a function of the fraction of rewired nodes, when does the diameter decrease most rapidly? Repeat the experiment for a 100-node network.

Model: Preferential Attachment. (Location: Models Library/Sample Models/Networks.) The simulation grows a network by adding nodes and edges according a simple rich-get-richer principle: the likelihood that a new node a will be connected to an existing one b is proportional to the *relative* degree of b (equals the ratio of the degree of b to twice the total number of edges in the current network).

9.4 Journal query.

Run the simulation for at least 1000 time steps—producing as many nodes. Describe the evolution of the degree-distribution. When, in terms of the number of nodes, does power law behavior become apparent? Where in the distribution—that is, for what degree values—is there the greatest deviation from a power law? Explanation? Estimate the power law's exponent. Recall that a power law has the form

$$y = x^m.$$

The exponent appears as the slope of a log-log plot:

$$\log y = \log x^m = m \log x.$$

In the model, you can mouse-over a point in the log-log plot to obtain its coordinates.

9.5 Journal query.

Briefly discuss how a preferential attachment process might apply to some type of real-world network.