Sample project topics Math 309

Bear in mind: The quality of a project is directly related to its having **substantive content** and a **well-defined**, **narrow focus**. The following are intended to be **suggestive**. Most topics need a narrowing of focus.

- Work out an example of a complex system—real or imagined—where the "whole is greater than the sum of parts" has clear application. Discuss how you can see this phenomenon in clear, quantitative terms. What are the parts? How do they "add up?" What's the whole?
- Discuss the claim that a large city has a fractal-like structure. Identify agents and interactions between them. What processes allow such a structure to emerge? Do fractal properties play a role in a city's functioning?
- Consider a single particle (atom, molecule, say) inside a container. Now add a second particle identical to the first. Describe a property of the 2-particle system that requires the activity of *both* particles. That is, a property that neither particle has individually. What about a property of a three-particle system that's not a property of any of the individual particles nor of any of the pairs of particles. Can this process continue indefinitely? Emergence depends on collective behavior, but how much of a collection of agents is enough?
- Consider or create a piece of art in which you see complex form or behavior. What are the agents in the piece and how do they interact? How does an awareness of emergent properties deepen your understanding of the work? You might focus on a particular style such as pointilism, futurism, cubism.
- Investigate the formation of *emergent structure* produced by bees building a hive (or a comparable phenomenon). What rules do the individual agents follow? How do they interact? What effect does a small change in the rules have on the structure that emerges?
- Gather data on the movements of huddles of emperor penguins. By prescribing rules for each bird to follow, devise an agent-based dynamical model that captures the penguins' motion. It might be a good idea to begin with a small number of agents and then try to increase the number.
- Select, modify, or create a *NetLogo* model and explore its behavior. Devise and conduct experiments that provide insight into the model's emergent properties.
- Set up a computer model that emits a flash (or flashes a dot) at several locations (say, at three points). How does the flash rate and sequence flashes affect what the eye perceives as the flashing occurs? Does the position of the points matter (line segment or triangle, in the case of three points)? What about periodicity or randomness in the sequence of flashes?

- Design a network that synchronizes a behavior (as in firefly flashing). What structures in the network are crucial for the synchronization? Simulating the behavior in a *NetLogo* model would be interesting.
- Experiment with various initial configurations using cellular automaton Rule 79. How does the system tend to behave after many iterations? By considering a small number of cells, try to account for the results.
- Explore "chromatic cellular automata" whose states are a color from a fixed set (three colors, say) and a neighborhood rule for determining the next state. It would be nice to see it simulated in *NetLogo* (or other system).
- Develop a *NetLogo* (or other system) model of a cellular automaton that generates concentric circles of states.
- Work out a *NetLogo* model of an evolutionary biological process in which interactions among agents conform to a game-theoretic payoff rule. You could give the interaction dynamics spatial properties—a model of an eco-space. How and where the interactions occur could be an adjustable parameter. Also, a method of mutation introduction is needed; parameters could specify when, where, and to what degree mutations occur.

  Simulate interactions over time and multiple generations. How do the populations of the respective phenotypes behave over time? How does the outcome depend on the initial conditions—number of mutants, mutation rate, etc.
- Investigate the dynamical behavior of a grid of springs that are connected to each other as well as to a boundary shape. (The investigation can be carried out by constructing a physical system, a virtual system—simulation, or by theoretical development.) Examples of grid arrangements are triangles, squares, or hexagons. Identify emergent properties or behavior that results from disturbing the grid away from equilibrium. Are there natural systems that behave in similar ways? What role does the density of springs play in the emergence of dynamical properties? How are emergent properties affected by making changes—small ones being the most interesting—in the boundary shape?