

Due: 12N Friday 31 October

Writing

- Using *concise and clear sentence*s, incorporate symbols, calculations, and, above all, illustrations into the text. Have an audience in mind. *Explain* what you're doing and why you're doing it.
- You may consult classmates and the instructor, but what you write should be *entirely your own work*.
- Typed work is appreciated. Hand-drawn figures are acceptable.

- 1) **Classifying polyhedra.** To classify polygons by type, we can use vertices and edges. There's only one type of polygon with n vertices and n edges. (Indeed, just one of these numbers is enough.)

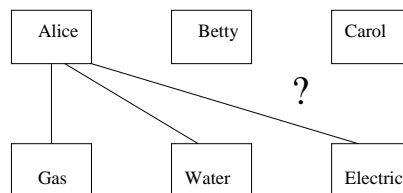
Can we use vertices, edges, and faces to classify spherical polyhedra by type? That is, given V , E , F such that $V - E + F = 2$, is there *at most* one type of polyhedron with V vertices, E edges, and F faces? Looking for simple examples to show that this doesn't happen might be a good first step.

Now, suppose we have three number $x, y, z \geq 4$ such that $x - y + z = 2$. Is there a polyhedron for which $V = x$, $E = y$, $F = z$?

- 2) **Polyhedra with seven or eight edges.** Can a spherical polyhedron have seven edges? If so, find all of the types. If not, prove it. Same question for eight edges.
- 3) **What can't make a spherical polyhedron** Use the Euler number to prove that there's no spherical polyhedron composed of triangular faces meeting six at *each* vertex. Note: it follows that no regular spherical polyhedron can have seven triangles at a vertex.

Generalize the argument to the following cases.

- k triangles per vertex where $k > 6$
 - of k quadrilaterals per vertex where $k > 4$
 - of k pentagons per vertex where $k > 3$
 - k n -gons per vertex where $k > 3$, $n > 5$.
- 4) **Five-faced polyhedra.** Two polyhedra P and Q are of the same *type* if they describe equivalent networks of vertices and edges. That is, you can turn P into Q by stretching and shrinking (but not cutting) edges.
- Use the euler number, polyhedron inequalities, and V , E , F numerology to describe *every type* of sphere-like polyhedra with five faces.
Note: Be careful not only to construct the constructible polyhedra, but to argue that they are the only constructible forms.
 - Use this result to show that you can't connect three houses to three utilities (gas, water, and electric) *without* having the connections cross.



- c) Suppose Alice, Betty, and Carol live on a torus (donut surface). Show that their houses *can* be connected to the three utilities without the connections crossing. Where does your argument above break down?

5) **Exploring frieze patterns**

- a) Design two frieze patterns that have the *same* symmetries, but look different in terms of the design.
- b) In addition to its fundamental translational symmetry, a frieze pattern can have one or more of four symmetries: rotational, reflection through a mirror *along* the strip, reflection through a mirror *across* the strip, and glide reflection. List the sixteen possible combinations of frieze symmetries. Which ones aren't actually possible to obtain? Which ones remain?

- 6) **Essence of a cube** Find a fundamental domain for the cube under its rotational symmetries and describe the quotient orbifold (give the orbifold code). Find a fundamental domain and quotient orbifold with reflective symmetries included. Do the same for the regular octahedron. Discuss how the two cases compare.

In-class presentations

- 1) **Look-alike polyhedra.** Recall what it means for the vertices of a polyhedron to “look the same.” Find a polyhedron in which all faces are congruent to one another and all vertices look the same, but the faces are not regular. Now find a polyhedron where all faces are regular (but, not the same shape) and every vertex looks the same. Try to find the *simplest* examples.
- 2) **Six-faced polyhedra.** Use the Euler number, the polyhedron inequality, and V, E, F numerology to describe *all* of the types of sphere-like polyhedra with six faces.
- Suggestion:* Consider how you can make a six-faced polyhedron out of a five-faced one.
- 3) **Polyhedral baseball.** Describe the symmetries of a baseball (ignore the stitches—consider only the seam formed by the two pieces of the cover). Now, build a polyhedron that has *exactly* the same symmetries as a baseball.
- 4) **Polyhedron inequalities.** In class we used the Euler number and relationships between faces, vertices and, edges to derive an inequality that describes a condition that the faces of a sphere-like polyhedron must satisfy:

$$3F_3 + 2F_4 + F_5 \geq 12.$$

- a) Is there a similar inequality that the vertices must satisfy? If so, what is it? If not, why not?
- b) Develop a polyhedron inequality—either for faces or vertices—for a torus-like polyhedron. How informative is it?