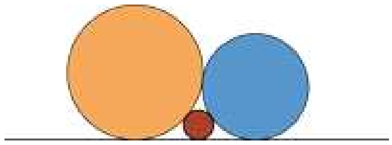
Math 355—Japanese temple problems[1]

"Devotees of math, evidently samurai, merchants, and farmers, would solve a wide variety of geometry problems, inscribe their efforts in delicately colored wooden tablets and hang the works under the roofs of religious buildings."

1) "Here is a simple problem that has survived on an 1824 tablet in Gumma Prefecture. The orange (largest) and blue (next largest) circles touch each other at one point and are tangent to the same line. The small red circle touches both of the larger circles and is also tangent to the same line. How are the radii of the three circles related?"

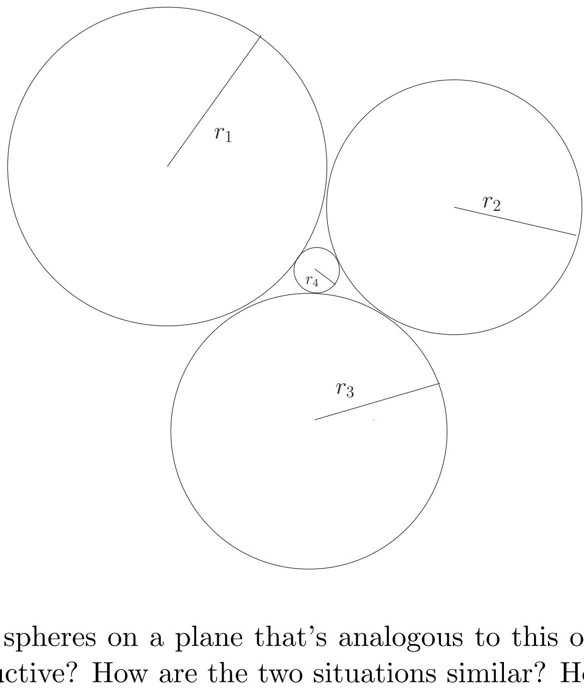
   Call the three radii r_1, r_2, r_3 and express r_3 in terms of r_1 and r_2.



2) Consider the situation when there are **three circles** that are mutually tangent. Fit a fourth circle that's tangent to the first three. How do you express the radius r_4 of the fourth circle in terms of the radii r_1, r_2 and r_3 of the other three?

   This isn't so easy. You might try the simpler cases where

$$r_1 = r_2 = r_3 \qquad \text{and} \qquad r_1 = r_2.$$



3) State a problem for spheres on a plane that's analogous to this one. Try to solve it. Is the circle problem instructive? How are the two situations similar? How do they differ?

4) Create your own temple problem. (You needn't know how to solve it.) Is it interesting? Does it extend to other dimensions? What makes a good problem, anyway?

[1] From *Scientific American* May 1998, pp. 85-91.

5) **Project idea** "Hidetoshi Fukagawa was so fascinated with this problem, which dates from 1798, that he built a wooden model of it. Let a large sphere be surrounded by 30 small, identical spheres, each of which touches its four small-sphere neighbors as well as the large sphere. How is the radius of the large sphere related to that of the small spheres?"

This problem is connected to the subject of polyhedra—something that we'll study later in the semester.