1 Exercises Math 444 Fall 2009

Though these won't be collected, you should write up solutions. Try to develop concise yet thorough arguments.

Reading

Chs. 1-4

From the text

Ch 3 Exercises 4
Ch 4 Exercises 5, 6, 7

1.1 Modular number systems

Take the positive integer n and consider two integers to be the same if they have the same remainder when divided by n. For example, let $n = 3$, then $1, 4, 7, \ldots, 3\,k + 1$ are *equivalent*.

1) Show that this way of identifying numbers is an equivalence relation on the integers $\mathbf{Z}$. The set of n equivalence classes is called $\mathbf{Z}_n$, the "integers mod n."
2) Define "addition mod n" in the obvious way—taking the remainder of the sum. Show that $\mathbf{Z}_n$ is a group under addition mod n. When does $\mathbf{Z}_n$ have proper subgroups?
3) Define "multiplication mod n" in the obvious way—taking the remainder of the product.
   a) Is $\mathbf{Z}_3 - \{0\}$ a group under multiplication mod n? Show that it is or isn't.
   b) Is $\mathbf{Z}_4 - \{0\}$ a group under multiplication mod n?
   c) What about $\mathbf{Z}_5 - \{0\}$? $\mathbf{Z}_6 - \{0\}$? What's the tricky issue here?
   d) When is $\mathbf{Z}_n - \{0\}$ a group under multiplication? Proof?

1.2 Reflection-rotation relations

Let R and M be rotational and reflective symmetries of a plane figure $\mathcal{F}$ respectively. Assume that $\mathcal{F}$ is *finite*; that is, $\mathcal{F}$ can be enclosed in a circle.

1) Give a geometric condition on R, M, and $\mathcal{F}$ that determines whether or not R and M commute. That is, under what conditions do we get

$$MRM = R?$$

   Hint: Since $\mathcal{F}$ is finite, what can you say about the center of R and the mirror of M?
2) Give a geometric condition that yields

$$RMR = M.$$

3) What happens in these cases if you don't assume that $\mathcal{F}$ is finite?

1.3 Double triangular pyramid

Consider two regular tetrahedra that are glued together at a face. They form a 6-faced polyhedron—call it P.

1) What are the **special points** on P?
2) Describe the group $\mathcal{R}$ of **rotational symmetries** of P. What's the order of the group? Is this a familiar group? (That is, have we seen it before?) List all of the subgroups of $\mathcal{R}$.
3) Describe P's group $\mathcal{S}$ of **symmetries generated by reflections**. What's the order of the group? What about subgroups?

Class presentations (28/30 Sep)

1) **1.1:** 1, 2, 3(a,b)
2) **1.2:** 1
3) **1.3:** 1, 2