Exam 1 Math 444 Fall 2009
Due: In class, Wednesday, 21 Oct

- Write concisely and clearly.
- *Ethic*: In completing this paper you should not consult other sources—excepting the text and instructor. The work represented by what you write and submit should be entirely your own.

1 A group that's a collection of sets

Let S be a set of things and let P be the set of subsets of S. For $A, B \in P$, define

$$A * B = ((S - A) \cap B) \cup (A \cap (S - B)).$$

a) Show that $(P, *)$ is a commutative group. What's the group identity? Given $A \subset S$, what's the inverse of A?

b) Consider the set

$$S = \{\text{Alice, Bob, Carol, Don, Erin, Frank, Gary, Harriot }\}$$

Using the set operation $*$ find the subgroup $(Q, *)$ of $(P, *)$ generated by the sets

$$\{\text{Alice, Bob}\}, \{\text{Carol, Don}\}, \{\text{Erin, Frank}\}, \{\text{Gary, Harriot}\}.$$

c) Express Q as a direct product of cyclic groups.

d) (Bonus) Describe Q in *abstract* terms; that is, in terms of generators and relations:

$$Q = \langle a, b, c, \ldots \mid a^? = e,\ b^? = e,\ \ldots \rangle.$$

2 Group multiplication tables

For each of the following cases, can the table be the multiplication table of a group? If not, give a reason. If so, show that it is.

a)

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	d	c	a	b
d	c	d	b	a

b)

	a	b	c	d
a	a	b	c	d
b	b	d	a	c
c	c	a	d	b
d	d	c	b	a

c)

	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	c	b	c

d) *Note*: Here, e does not signify the identity element. It's just an abstract element.

	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	a	e	f	d
c	c	d	b	f	a	e
d	d	e	f	a	b	c
e	e	f	d	b	c	a
f	f	a	e	c	d	b

3 Groups whose elements have order two

Suppose that G is a group in which every non-identity element has order two. Show that G is commutative.

4 Cyclic group properties

Consider $\mathbf{Z}_n = \{0, 1, \ldots, n-1\}$.

a) Show that an element k is a generator of $\mathbf{Z}_n$ if and only if k and n are relatively prime.
b) Is every subgroup of $\mathbf{Z}_n$ cyclic? If so, give a proof. If not, provide an example.