Math 451 Spring 2009

Lab 2: Curvature of surfaces of revolution

Reminders

- Save your work *every time* you enter new material in a cell and *before* you send it to the kernel for evaluation.
- Your submission—a *single* worksheet called *yourname_lab#.mw*—should contain responses to the **Worksheet exercises**. Every cell of your notebook should have, at least, a preceding comment that describes the cell's content and what the outcome means.
- Remember the `Maple Help` facility accessible in the `Help` menu.
- *Maple* code appears in `typewriter font`.

Tasks

1) Write a procedure that will display a surface of revolution.
2) Compute the curvature of a curve produced by cutting the surface with a special plane.

Plotting a surface of revolution

In standard (x, y, z) coordinates, let $z = f(x)$ be a function defined in the $xz-plane$. After revolving the graph of f about the z axis, you can parametrize the resulting surface in polar terms by

$$(s, t) \longrightarrow (s\,\cos t, s\,\sin t, f(s))$$

where t is the angle relative to the x-axis.

Alternatively, you can express the surface as the graph of a function $g(x, y$:

$$(x, y) \longrightarrow (x, y, f(r)) \qquad \text{where } r = \sqrt{x^2 + y^2}.$$

Worksheet exercises

Using each method described above, use `plot3d` to create the plot the surface S obtained by revolving the graph of $z = 1/x$ about the z-axis. Recall that

```
plots[display]([plotname1,plotname2,...])
```

displays a plot. (Note: you have to be careful when selecting variable ranges if they contain a place where the function defining the surface blows up—in this case, at $(0, 0.)$

A special plane

Consider the point $(1, 0, 1)$ on the surface S obtained above. Fixing $y = 0$ gives a curve $(x, 0, f(x))$ along S in the xz-plane. Differentiating the curve, relative to the parameter x, at $x = 1$ gives a vector N tangent to S at $(1, 0, 1)$. Cutting S with a plane K perpendicular to N that passes though $(1, 0, 1)$ yields a curve in K.

Worksheet exercises

1) Compute N.
2) Compute the equation of K and parametrize the plane in x and y.
3) Plot K in the same graphic as S.
4) Include a plot of the z-axis that extends far enough to intersect K. (You can do this with a parametric plot of (x, x, z) by setting the range of x to be from 0 to 0.)

The curve formed by the plane and the surface.

We can use the coordinates x and y to define coordinates s and t *on the plane* K. Think of an s-axis and t-axis where the s-axis is formed by the intersection of the xz-plane and K while the t-axis coincides with the y axis.

Worksheet exercises

1) What are the change-of-coordinate functions

$$s = F(x, y) \qquad \text{and} \qquad t = G(x, y)$$

   that turn x-y coordinates on the xy-plane into s-t coordinates on K?
2) Treat t as an independent variable and parametrize the plane curve $C = K \cap S$ near $(1, 0, 1)$ as $(s(t), t)$. What are the s-t coordinates of the point $(1, 0, 1)$?
3) Use the curvature function (Lab 1) to compute the curvature in K of C at $(1, 0, 1)$.
4) Recall that the radius of curvature is the reciprocal of the curvature. Compute the radius of curvature and then, in the s-t coordinates, the center of curvature of C at $(1, 0, 1)$.
5) Now compute the corresponding tangent vector and normal plane at the point $(2, 0, \frac{1}{2})$. Use the same procedure as above to compute the center of curvature of the curve obtained by cutting S with the normal plane. (Note: You need to make a similar change of coordinates from x-y to the normal plane.
6) In light of the two results for the locations of the respective centers of curvature, form a conjecture regarding the center of curvature for the normal plane of this type at any point on S.
7) **Bonus:** Give a heuristic proof of your conjecture for any surface of revolution.

   Suggestion: Think about fitting an osculating circle to the curve at the point in question. What is the instantaneous motion of the point as the surface is swept out by revolution?