## Sample project topics Math 451 Spring 2009

**Bear in mind:** The quality of a project is directly related to its having **substantive content** and a **well-defined**, **narrow focus**. The following are intended to be **suggestive**. Most topics need a narrowing of focus. Computer graphics and experiments might help to illuminate some of the ideas and results.

- At each point of a surface you can follow the directions of the principal curvatures. Doing so produces two systems of orthogonal curves on the surface. Develop the basic theory of these "lines of curvature." You might consider the case of an ellipsoid as a means to work out some of the interesting properties—such as their being the intersection of the ellipsoid with families of orthogonal surfaces—with that arise.

  *Source*: Hilbert and Cohn-Vossen, *Geometry and the Imagination*

- An *umbilic point* on a surface is one where the directional curvature is the same in all directions—as on the plane or sphere. On an ellipsoid most points aren't umbilic, but there are some special places that are. Discuss how to find these points and their connection to lines of curvature.

  *Source*: Hilbert and Cohn-Vossen, *Geometry and the Imagination*

- A sphere has the property that it can't be bent—that is, deformed in a way that doesn't distort distance? However, if you remove an arbitrarily small patch from the sphere, the remaining surface is bendable. Investigate these phenomena and develop arguments for these claims.

  *Source*: Hilbert and Cohn-Vossen, *Geometry and the Imagination*

- An important idea in differential geometry concerns moving a tangent vector on a surface $S$ in such a way that all the vectors that arise are tangent to $S$ and are *parallel* in to the intrinsic geometry of the surface. (This does not mean parallel as vectors in $\mathbf{R}^3$. It might be that none of the vectors tangent to $S$ at one point are parallel as vectors in $\mathbf{R}^3$ to a tangent vector at another point.) Investigate the notion of *parallel transport* of vectors which supplies an intrinsically meaningful definition 'parallel.'

  Use the notion of parallel transport on the sphere to describe the behavior of a Foucault pendulum.

  *Sources*: Oprea, *Differential Geometry and its Applications*

  Oprea, *Geometry and the Foucault pendulum* (Amer. Math. Monthly, June-July 1995)

- The cycloid has the remarkable property that if an object moves along the curve free of friction under a constant force—such as gravity over a small distance, it takes the same amount of time to reach the bottom regardless of the height at which it starts. Establish that the cycloid is the "tautochrone" (meaning "same time").

  *Source*: McCleary, *Geometry from a Differential Viewpoint*

- Another notion of curvature of a surface at a point is *mean curvature* which is the arithmetic average of the principal curvatures. Given a closed loop $C$ without self-intersections, you can ask what surface $M_C$ has $C$ for a boundary and has minimum surface area. (This property of being minimal is physically realized by forming a closed loop of wire and placing a soap-film surface within the loop.)

  If $C$ is planar, the *minimal surface* $M_C$ is just the piece of the plane bounded by $C$. In this case, the mean curvature of $M_C$ is zero. The mean curvature vanishes for a surface that's minimal when $C$ is not planar as well. Examine the elements of the theory of minimal surfaces and establish this basic result. There's also the question of whether there's a unique minimal surface for a given $C$.

  *Source*: Courant and Robbins, *What is Mathematics?*
- Find a problem or piece of theory to solve or develop. There are many texts to consult. Here are a few.

  - O'Neill, *Elementary Differential Geometry*
  - Oprea, *Differential Geometry and its Applications*
  - Pressley, *Elementary Differential Geometry*
  - Thorpe, *Elementary Topics in Differential Geometry*